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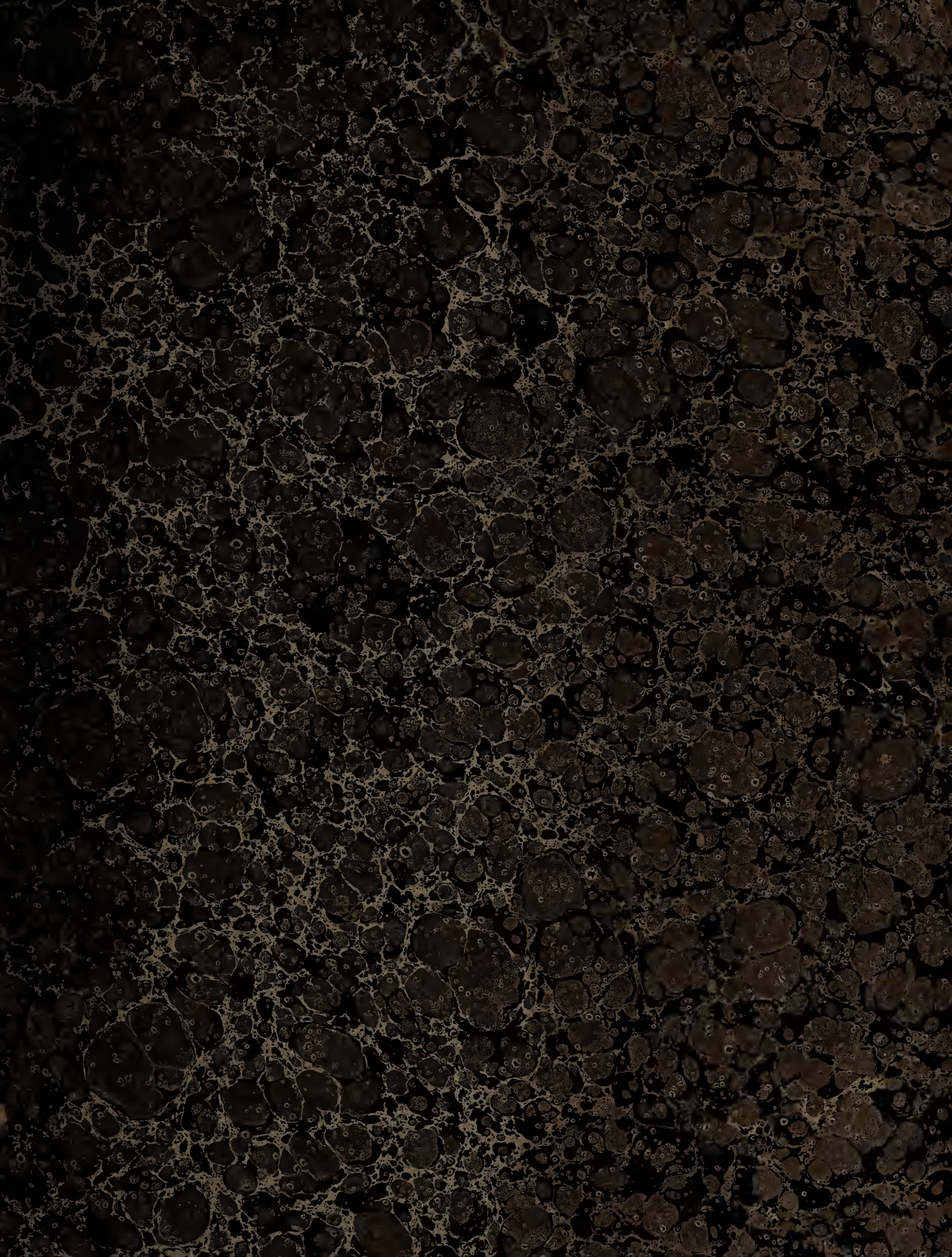


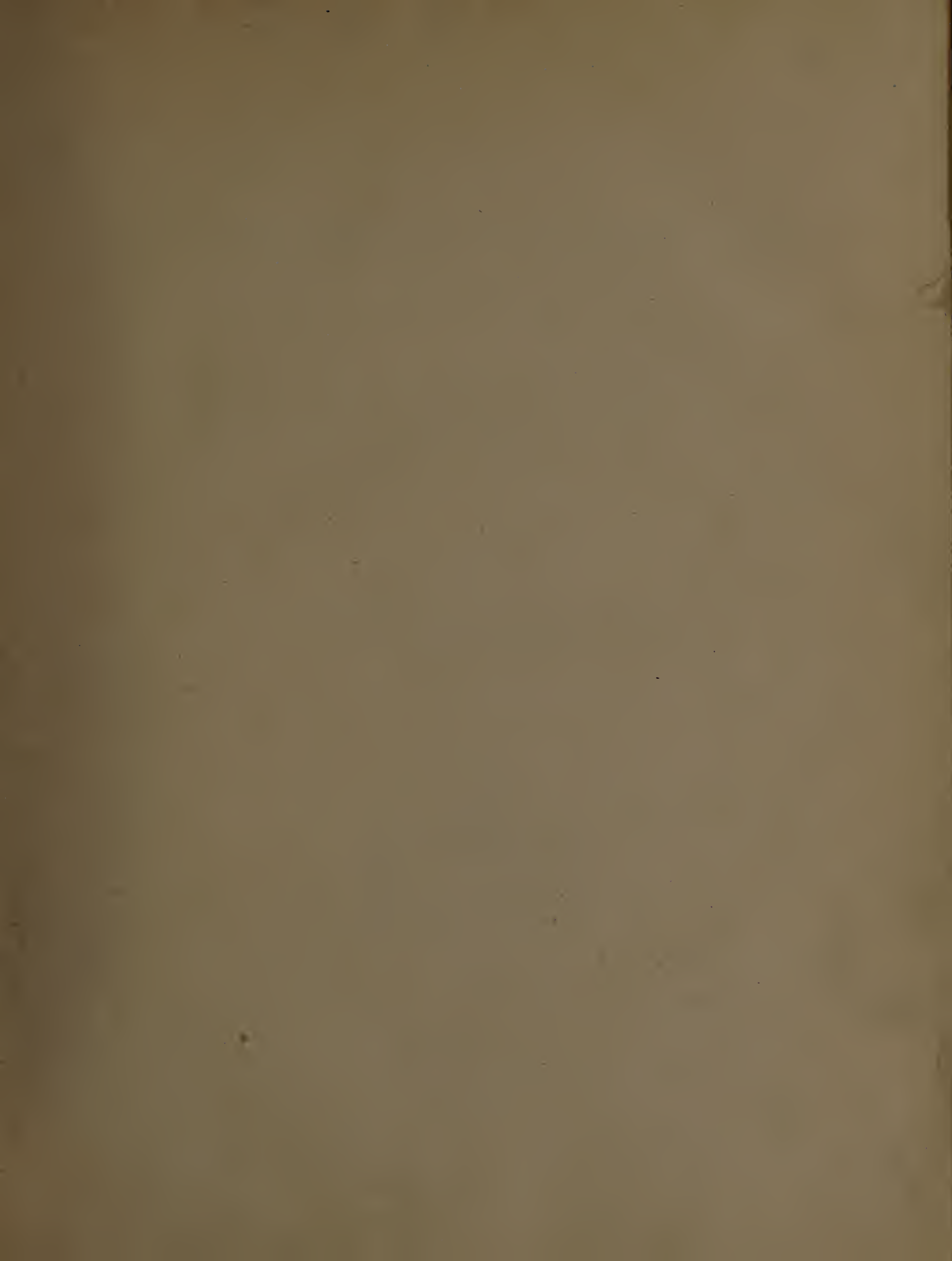
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Presented by

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Nº 31014





ON

THE EARLY HISTORY OF THE SIGNS + AND -.

FROM THE TRANSACTIONS OF THE CAMBRIDGE PHILOSOPHICAL SOCIETY,
VOL. XI. PART I.

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IX. *On the Early History of the Signs + and -*. By AUGUSTUS DE MORGAN, F.R.A.S., of Trinity College, Professor of Mathematics in University College London.

[Read November 28, 1864.]

Two or three years ago my friend Mr John Bellingham Inglis, whose collection of old books swarms with rarities, gave me a work on arithmetic printed in 1489, with the remark that books which he could not use himself he liked to put into the hands of those who could use them. By this application of a principle not universal among book-collectors, he has added forty years to the known age of the signs + and -, besides bringing to light some curious presumptions as to their origin, and as to their having a still older date.

The writer of this book is John Widman of Eger. I can find no account of him in ordinary biographical sources. He writes in a clear and able manner, and gives presumptions of originality, though it is difficult to verify this upon an author of his date. Very few older works on arithmetic will be found: the only one of which I can give notice which has a date is given by Hain as a *Rechenpuchlein*—the German of this period uses *b* and *p* &c. as convertible letters—printed at Babenberg in 1483.

It may be a question whether this John Widman is the early, perhaps the earliest, writer on the *morbis gallicus*, in the work entitled 'Tractatus clarissimi medecinarum doctoris Johannis Widman, dicti Meichinger, de pustulis et morbo qui vulgato nomine *mal de franxos* appellatur,' 4to. *sine loco*, 1497. Mr Inglis has two copies, one imperfect, the other *pluperfect*, having two more leaves than the eight which the bibliographers give it. This small tract is mentioned by medical bibliographers from Lipenius to Haller. John Widman wrote other medical works after 1500, which are described, with discussion, by Astruc, *De Morbis Venereis*, 4to. 1740, (Vol. II. pp. 556—564). I incline to the opinion that the two Widmans are only one. The physician of that day was so often the mathematician that there is the same sort of presumption which would arise in our day if a writer on *physics* had the same name as a mathematician. It should be noted that the physician Widman passed under the Latin form *Salicetus*.

The work on arithmetic is 'Behēde vnd hubsche Rechenung auff allen kauffmanschaft.' At the end is 'Gedruckt In der furstlichen Stath Leipczick durch Conradū Racheloffen Im 1489 Jare.' The work is in octavo, beautifully printed on the thick and rough paper called *pergamenus* from its approach in appearance to parchment. The preface ends with a new-year's wish*, and the words 'Gegeben tzu leyptzick zcum nawen Jare der Weniger zcal Nach Christi geburt Im neunvndachczigsten.' The first words of the preface are *Johannes Widman von Eger*; and he speaks as an author.

The work is mentioned by Panzer, Hain, Ebert, Murhard, &c.: Hain calls it anonymous. Panzer may have owed his knowledge of it to C. B. Lengnich of Dantzic, who, as appears by an inscription, in 1788 gave him the copy which I now possess. A copy was sold in England at Kloss's sale, which may be the one I have; most likely there is one in the Dresden library. There are several reprints. My friend Mr John Thomas Graves has editions of Pforzheim, 1508, and Hagenau, 1519. Ebert mentions an edition of Augsburg, 1526, 8vo. It is important to note that the work was widely known.

I was led to look through this work by finding elsewhere uses of + and −, about 1525, different from that made by Chr. Rudolff at the same time. As the work is rare in every edition, I shall make full extract from the most important places in which the signs occur. As there is no paging—there are not even catch-words, and only one page in a sheet has signature—I must refer by signature and page of the sheet.

(l. 11). "Als in diesē exēpel 16 elln̄ pr 9 fl $\frac{1}{3}$ vn̄ $\frac{1}{4}$ + $\frac{1}{5}$ eyNSS fl wy kūmē 36 elln̄

* The *etc.* was printed by a *t* brought down to admit a circumflex above it, followed by a *c*; the *t* resembling the *r* of written hand. This *tc*, as I may print it, seems to have been used for *xc*, meaning 90, as a kind of practical joke upon the reader, in Leipsic works of 1490. Widman's work must have given the new year's wish for 1490, since the printing took place in 1489: and that wish is 'liebhaber tzu eynem geluckse ligen nawen Jare *tc*?' In another Leipsic work of the same year, the *Algorithmus Linealis*, the colophon is 'Impressum Liptzk pr melchiorē Lotter, Anno *tc*.'

It is not without significance that while later Germans write Widemann, Wiedemann, Widmann, &c. both Von Eger and Meichinger, be they one or two, write themselves nothing but *Widman*. I have found one biographer who mentions both, the minute and laborious Zedler: but he gives nothing except what might be collected from the books, and shews that he has not himself seen the books, and that he follows incorrect descriptions. The following is the account of Widman von Eger.

Wiedemann oder Wiedemaun (Johann) von Eger in Böhmen gebürtig, studirte zu Leipzig, nahm auch daselbst die Magister Würde an, und hielte nach diesem starck Collegia mit grossem Beyfall, wendete sich endlich wieder nach seiner Vaterstadt. Er florirte zu ende des 15 und zu Anfang des 16 Jahrhunderts. Man hat von ihm

1. *Algorithmos integrorum cum probis* Lib. i.
2. *Algorithmos minutiarum vulgarium* Lib. i.

3. *Algorithmos minutiarum physicarum*, Lib. i.

4. *Algorithmos proportionum*, Lib. v.

5. *Algorithmos lineales*, Lib. i.

6. *Libellum summarium totius Arithmeticae*, so von Siegmund Altmann ins Deutsche übersetzt worden, nebst andern mehr.

Mader Centur. Scriptor.

On Meichinger is quoted Scheuchzer, *Bibl. Scrip. Hist. Nat.*; Manget, *Bibl. Script. Medic.*; Zauber, *Zusatze der Historie der Land-Charten* p. 115.

Mader has misled Zedler by a description which can only be paralleled by attributing to Moses the authorship of Genesis, Exodus, &c. and the Pentateuch as a sixth work. The *libellum summarium* before me is well enough described in the five preceding heads, except only that it would be difficult to point out five books in proportion. The *algorithmus linealis* is a small collection of geometrical applications at the end. Zedler's separation of the authors is the duty of a biographer towards different second surnames, especially of a biographer who does not know how nearly the periods of publication of the two sets of works begin and end together. Probably Zedler never saw the two names together until the proof-sheet was before him. His alphabet is vastly greater in number of words than that of any preceding encyclopedist: his medical and mathematical writers must have come from different sources at different times.

machss also Addir $\frac{1}{3}$ vñ $\frac{1}{4}$ vñ $\frac{1}{5}$ zu sāmen kumpt $\frac{47}{60}$ eynss fl " Here the + is probably a sign to which the writer was accustomed, but which he did not then intend to print, since *und* occurs both before and after. The printing may have taken place at a distance, or the author may not have cared to correct the lapse in proof: both things were common.

Be it which it may, the above passage must stand, for the present, as the earliest hitherto quoted in which the sign + appears.

(l. 12, 13, 14) Veygen.

¶ Itm Eyner kaufft 13 lagel veygen vñ nympt ye 1 *cl* [100 pounds; an abbreviated type is used for *cl*] pr 4 fl $\frac{1}{2}$ [misprint for $\frac{1}{8}$] ort Und wigt itliche lagel als dan hye nochulget. vñ ich wolt wissen was an der sum brecht

C zentner	{	4 + 5	Wiltu dass
		4 - 17	wyszen ad
		3 + 36	dess gleichñ
		4 - 19	Szo sum-
		3 + 44	mir die <i>cl</i>
		3 + 22	vnd lb vñ
		3 - 11	lb was—ist
		3 + 50	dz ist mī ⁹
		4 - 16	dz secz besū
		3 + 44	der vñ wer
		3 + 29	dē 4539
		3 - 12	lb (So du
		3 + 9	die <i>cl</i> zcu lb

gemacht hast vnnd das + das ist mer dar zu addirest) vnd 75 min⁹ Nu solt du fur holcz abschlahñ albeg fur eyn lagel 24 lb vñ dz ist 13 mol 24 vñ macht 312 lb dar zu addir dz—dz ist 75 lb vnnd werden 387 Die subtrahir vonn 4539 Unnd pleybñ 4152 lb Nu sprich 100 lb das ist 1 *cl* pr 4 fl $\frac{1}{8}$ wie kummen 4152 lb vnd kūmen 171 fl 5 ss 4 hlr $\frac{4}{5}$ Un̄ ist recht gemacht.

Before I proceed to comment on this question, I observe that the invention of signs of operation did not suggest itself to the arithmeticians of the time. It will be said that + and - are here to dispute my assertion. Put these out of sight for a moment, the question I shall raise being whether + and - did not come to the arithmetician *from the warehouse*. There is another case which shows us that the arithmetician's own *directive* symbols never became signs of *operation*, though in almost universal use for a century.

In the book before me I find the earliest use I have met with of the *indicating lines* by which a fourth proportional to three fractions is found.

The first instance is (l. 12)

$$\frac{32}{1} \times \frac{69}{4} \text{ — } \frac{3}{1},$$

which guides the choice of factors in 1.69.3 divided by 32.4.1. The guide-lines seem to have been suggested by the complication of the case: for Widman does not,

as others did after him, use $\times$ alone in a division, and $=$ alone in a multiplication. This notation was used also by Pacioli, Tartaglia, Orontius Fineus, John Weber, Cataneo, Urstisius, Perez de Moya, Peverone, D'Ortega, Unicornio, Ghaligai, Huswirt, Sfortunati, Forestani, Chr. Rudolff and Stifelius: I take them as they come to hand. But no one of them ever used $\times$ as a sign of division, nor $=$ as a sign of multiplication: the lines guided the details of these operations, but never symbolised the operations themselves. Had one of the above writers made the step of allowing $\times$ and $=$ to symbolise *direction* to divide and to multiply, not merely signs of guidance in division and multiplication *previously directed*, I doubt not that the notation would have become permanent. In order to give a notion of the slowness of growth of symbolic language which characterised the period 1450—1550, I may satirise it as follows. If a white post had always indicated, when met with in the road, that the last turning was wrongly made, so that the traveller must go back to the junction, and take the other turning, those of whom I speak would never have arrived at a distinct notion that the post was a direction to turn back; they would have allowed it to remain an announcement of error, and would always have made an inference of retrogression.

In his fractional questions, Widman is not symbolist enough to separate $\times$ and $=$ for their separate uses: and his combination is introduced abruptly, without explanation. I conclude that he is not the inventor. He had used only one symbol before he came to $\times =$, and it was $\times :$ but it was that mysterious cross, even yet known to school-boys, which took the four digits arising from the casting out of the nines, and decided the question of immediate play or another trial at the loathsome figures. He also gives verifications by casting out the sevens. He gets as far as sheet k. 11 without any other symbol of guidance, then brings forward $\times =$ without explanation, and produces $+$ and $-$ just as abruptly seventeen pages afterwards.

The mode of arrival at $\times =$ is quite manifest: let us now examine the example quoted above. The *lagel* is a cask, chest, or other wooden receptacle: of these there are 13, at $4\frac{1}{2}$ fl per 100 lb. with tare of 24 lb. for the weight of each lagel. The florin is 20 ss (shillings, stivers, as may be) of 12 heller each. The chests are weighed by centners of 100 lb. each, and the run of the chests being from 3 to 4 centners, the obvious plan is to put three or four centners into the scale of the weights, and make the balance by pounds in that scale, or in the scale of the goods, as wanted. The first chest wants *more*, and is 4c. 5lb: the second wants *less*, and is 4c. all but 17lb. It will fully appear that $+$ and $-$ are *plus* and *minus*, from which addition and subtraction follow by *inference*. This presentation of data is not the doing of the arithmetician, as such: it seems to be served up direct from the warehouse. It may be suspected that the nearest number of centners would be placed in the scale by guess, and separate record would be made of the overplus or underminus—if this last word may be allowed. It may be suspected that $+$ and $-$ were warehouse marks, annexed to the entry of the weights for distinction, perhaps painted or chalked on the chests. The words *plus* and *minus* are very old commercial terms, and the word *overplus*—which occurs twice in the authorised version of the Old Testament, Levit. xxv. 27 and 1 Maccab. x. 41,

and is also in the preceding versions—is of commercial origin. In all probability the use of the *nearest* large weight would be a refinement, and the usual plan would be to have an overplus: the word underminus would thus be avoided. All this conjecture only indicates the sort of inquiry which must be made into earlier print and manuscript.

(l. 14). ¶ Itm̄ 1 Sack pfeffer wigt 2 cl $\frac{1}{2}$ - lb vn̄ ist 1 lb kaufft wordē pr 8 ss - 3 hlr vn̄ sol fur den sack abschlahn 3 lb + $\frac{3}{4}$

The warehouse plan does not occur in one question only.

(m. 8) Seiffen

¶ Itē Eyner kaufft 4 lagel seyffn̄ wigt die erst 4 cl - 63 lb Die ander 3 cl + 24 lb. Die drit 3 cl - 2 lb Die viert 4 cl + 1 lb Und kost ye 1 cl 5 fl 18 ss 1 hlr vnd geth ab fur das holcz ye fur 1 cl 12 lb Machsz alss obn̄ vnd kūpt 70 fl 13 ss 2 hlr $\frac{82}{125}$.

(o. 13) The heading *Regula augmenti + decrementi* is probably a sort of joke, a use of + for logical aggregation: the *regula augmenti* alone is just before. The rule is that for solution of $ax - b = cx + d$, and the following is the example given.

Anis

¶ Itm̄ Eyner will eyn sack mit Anisz kauffen Und wen er vor itlichss lb 12 d giebt szo pleiben ym vberig 37 d Szo er aber fur 1 lb 15 d giebt so zu rint ym 44 d Nu ist die frag wie vil der sagkge wegen hab vnd wie vil er gelcz gehabt hab Wiltu das wissen und des gleichen So machss nach der Regel also subtrahir 12 von 15 pleyben 3. vnd das ist der teyler darnach addir + und - zcu sam wirt 81 die diuidir mit 3 kummen 27 lb vnd so vil hat der sack gewegē mit dem Anis Nu multiplicir 12 mit 27. vn̄ addir 37 darzu Ader multiplicir 15 mit 27 und subtrahir 44 kūmen 361 vnnd szo vil ist des gelcz gewesen vnd ist gemacht.

The equation is $12x + 37 = 15x - 44$: 'addir + und -' is to add the *more* and *less*, 37 and 44: + and - stand for the quantities themselves.

Some of the bibliographers notice that Widman gives the chain-rule: he does so, and under symbols which look more like a chain than the modern arrangement. The chain (v. 10) is

7	9	12	13	8	18	30
	\	x	x	x	x	x
	8	11	15	10	5	4

and the data are—7 wyener gelten 9 linczer vnd 8 linczer geltn̄ 11 passawer vnd 12 passawer geltn̄ 13 vilshofer vnd 15 vilshofer geltn̄ 10 regensperger vnd 8 regensperger geltn̄ 18 neumercker vn̄ 5 neumercker geltn̄ 4 nurr̄mberger wie vil kummen wiener vmb 30 vur̄mber.

The answer is $7 \cdot 8 \cdot 12 \cdot 15 \cdot 8 \cdot 5 \cdot 30$ divided by $9 \cdot 11 \cdot 13 \cdot 10 \cdot 18 \cdot 4$. It is strange that those who detected the chain-rule should never have noticed + and -, which occur in force six pages before: they probably thought that + and - were Noachian.

There is yet another use of the signs, which continued for a century. In the Elcataym, or *regula falsi*, it became an established convention that $a + b$ should mean that the assumption of a gives b too much of answer. For this rule I may refer to Dr Peacock's historical article.

(C. 1). ¶ It̄m Eyner hat Ingwer vnd pfeffer 80 lb vnd giebt 5 lb Ingwerss vmb 1 fl Und 8 lb pfefferss auch vmb 1 fl vnnd ist itlichss 40 lb Nu ist die frag wie vil lb fur 2 fl kūmen Ingwer vnd pfeffer vñ eyyss als vil als dess andern vntereynād Mahc̄ss also vnd secz er geb ym 12 lb vmb 12 fl vntereynāder Sprich 80 lb gelten 13 fl was geltn̄ 12 lb pfeffer vñ Ingwer vnd kumpt $12\frac{9}{20}$ vnd leugt minus $\frac{1}{20}$ wan es sollen 2 fl kummen seȳ Darnach secz Er geb im 13 lb fur 12 fl Sprich 80 lb kosten 13 fl was kostē 13 lb facit $2\frac{9}{80}$ und stet also tc

$$\begin{array}{r} 12 - \frac{1}{20} \\ \times \\ 13 + \frac{9}{80} \end{array} \quad \begin{array}{l} \text{Nu machss} \\ \text{noch der} \\ \text{Regel vnd} \end{array} \quad \frac{260}{1600}$$

kumpt $12\frac{4}{13}$ lb vnd ist recht gemacht.

Here 12 gives $\frac{1}{20}$ too little, and 13 gives $\frac{9}{80}$ too much. And *unlike signs give addition and like signs give subtraction*: and the answer is, in modern signs,

$$12 \times \frac{9}{80} + 13 \times \frac{1}{20} \text{ divided by } \frac{9}{80} + \frac{1}{20}, \text{ or } 12\frac{4}{13}.$$

This form of solution, with the data disposed about the cross $\times$, runs through a host of writers, who generally use *p* and *m* for the indications of over and under solution. Some Germans, beginning, so far as yet known, with Widman, use + and -; among whom is Adam Riese and a writer printed by Apian. The first work is *Rechenung auff der linihen...* Erffordt, 1525, 8vo; and the indication of + and - is very faint. The second is *Eyn Newe... unterweysung aller Kauffmanss Rechnung...* Ingoldstadt, 1527, 8vo. The rule is as follows.

Laut der Regel.

Setz fur dich eine bequeme zall, Welche nach laut der auffgab alle zufellige addition oder subtraction erleiden kan, vnnd operir do mit als wer sie die gerechte zall. Wann du durch die operation kummest zu dem endt so gibt dir die selbige zall das recht facit, zu zeitten meher oder minder, was zu vil ist vermerck mit dem zeichen plus +. Ist zu wenig, mit dem zeichenn minus -. Darnach nym fur dich aber eine andere zall, die auch gleich, dermassen alle stuck d' auffgab erleiden mag. Die lügen vorzeichen auch mit dem zeichen + oder -. Wan die zwo lügen gleiche benennung haben als + vnnd +, oder - und -, so subtrahir die klenner lügen von der grosten, das rest heist der gemain tailer. Sein sie aber mit beden zaychen denominirt so addir sie zu sammen die summa bedeut auch den gemainen tailer. was weiter darinne zu wissen ist wirstu aus den nachgesetzten exempeln genugsam vornemen

$$\begin{array}{cc} + & S & - \\ + & & - \\ \text{Subtrahir} & & \end{array} \quad \begin{array}{cc} + & A & - \\ - & & + \\ \text{Addir} & & \end{array}$$

Some writers detached the + and - and made them little appendages to the cross. This was done by Recorde in his *Ground of Artes* (first published in 1540), as I suppose from John Dee's edition of 1561, the earliest I know of. Tonstall (1522), who is not given to symbolic expression, uses *P* and *M*, as did all but the few Germans I have alluded to,

Recorde, and Orontius Fineus, whom I have no doubt Recorde followed. The rule of signs, that more and more, less and less, require subtraction, while more and less, less and more, require addition, are presented by Tonstall in verse, from the muse of no less a person than his friend Sir Thomas More, to whom the work is dedicated. 'Morus, rogatu nostro, redegit eas in hoc carmen.

A plure deme plusculum.
 Minus minori subtrahe.
 Pluri minus conjungito.
 Atque ad minus plus adjice.'

In favour of the warehouse theory it may be added that Widman is very chary of the use of the signs + and - throughout the part of his work which treats of fancy problems or of the higher rules of commerce: it may be that he was little accustomed to the signs except in the class of problems in which they reached him as data. If they were marks invented in business, the most probable surmise is that they have no deeper origin than the notion of +, two marks, suggesting more than -, one mark. Unless indeed, which is possible, some very old convention should connect + with the scale of weights, and - with the scale of goods. If the signs be really, in the first instance, symbolic of *plus* and *minus*, which may have been even if they took their rise in actual business, then abbreviation of *p* and *m* is the most probable origin. The $\mathfrak{P}$ of old letter, which still retains some currency in actual business, and is therefore a type of our day, easily contracts into +; and - is the limiting state of *m* in rapid writing, even in our own day, in which the final *m* often becomes a mere line. The ingenious supposition of an abbreviation of &, the symbol for *et*, must be abandoned, since + certainly meant *more*, independently of the direction to get the *more* by addition.

The work in which + and - were applied to algebra is taken to be the first edition of Christopher Rudolff's *Die Coss*, 1524. This first edition is lost: no copy of it is known to exist. A Latin translation, made in 1540, is said to exist in the Imperial Library at Paris, referred to as MSS 365 m. 4, and as 7365 of the Latin Manuscripts, 4to. No account has been given either of the contents of this manuscript or of their amount of agreement with the second edition (1554), by Michael Stifel, or Stifelius. This editor very distinctly maintains that his edition is a real second edition, with augmentation (specified in character) from Rudolff's papers; and he represents the original edition as having come to bear a very high price. And Rudolff himself, in the preface of his *Künstliche Rechnung*, 1526, represents himself as having 'im fünff unnd zwentzigsten jar'—that is, in 1525—printed 'ein Rechenbuch die aller künstlichsten Regel Algebre beschlisent'. Allowing that the probability is in favour of Stifel's edition being no more than an augmentation, as described, of that which preceded, there remains a fair possibility that Rudolff's first work may have been far below what it became in the editor's hands. Stifel—not Stifels, nor Stiffel, nor Stiefel—was so singular a man that his notions of editorship may have been of any amount of excentricity. In his appendix he does various other things as strange as the one by which he is well known, namely, his translation of Leo Decimus into 666 by dismissing *M* as the initial of *mysterium* and bringing in X from Leo X. His craziness is of the second order: for if the usual state of mind

in which 666 is made a prophecy of infamy to one or another person be called perfect soundness, then Stifel has still an unsoundness of the first order. I cannot tell what to make of the assertions of so curious a writer: and I shall not feel satisfied about the first edition of Rudolff until either a copy is produced, or a full account of the Latin manuscript is published.

The point to which I especially refer is this. We know that the inventors of our symbols attached very little importance to them: and would have stared with wonder if they had been told that these trumpery tricks of abbreviation would one day have a philosophy of their own, and would make inquirers curious about their origin.

If Rudolff had made some use of $+$ and $-$, without such extensive and permanent introduction as Stifel himself made in his *Arithmetica Integra* of 1544, this same Stifel, thinking little of the new signs as matter of merit, might very easily infuse into Rudolff's work such a quantity of use of $+$ and $-$ as would bring Rudolff into the apparent position of the first user on a large scale. And this without the least notion of his giving anything, or making any serious alteration in the character of the work. The mere invention and early use of the signs being now thrown back upon Widman at latest, and probably on his predecessors, the next question is, Who first saw the place which the abbreviations were to occupy, and gave them that place, so as to bring out their force and effect. Either Rudolff or Stifel, is the answer as yet: if Stifel found in Rudolff only such moderate use as Widman had made, and extended that use by his own alterations and insertions, it may be that his own work of 1544 is the first which made algebra look like what it now is in the use of these symbols.

In the *Künstliche Rechnung* of 1526, Rudolff does not show a single mention or use of $+$ and $-$: his signs are $\times$ as a pair of guides in the addition of fractions, and $\times =$ in the rule of three. If he had had his head as full of $+$ and $-$ as a man must have had who had published in 1525 a *Die Coss* like that of 1554, this would have been very strange.

There is another presumption which has some weight. Rudolff, as we have seen, owns to a book of 1525 which contains the *rules* of algebra: this by itself is no strong evidence against the book containing reasoned exemplifications. But Stifel informs us that Rudolff had been charged with not understanding his own rules, and with having taken them from a Vienna manuscript. This charge seems to indicate that only rules were given: or at least that the work falls very far below the *Die Coss* which issued from the hands of Stifel.

There is an account of Stifel's edition in Terquem's *Bulletin de Bibliographie*, &c. May and June 1855, pp. 71—83: this is reprinted in Tortolini's *Annali* &c. Vol. VIII. Sept. 1857. All the copies examined have the title-date 1571, and the colophon-date 1554; whence it has been inferred that the printed book lay unissued for 17 years. No one seems to have thought of the possibility of a new title-page. The descriptions of books were originally placed at the end: there was a title, which it would now be convenient to call an *incipit*, at the beginning of the first page; and this often not so much as a *heading*, but only a first sentence. The title-page was a subsequent invention; the *fly*-title was added long after. At the time I write of, the title-page was not firmly settled to be a part of the book: it was looked upon much as we look upon the binder's title at the back. In the case before us

the original title-page exists. My friend Mr John Thomas Graves, whose mathematical library is of the first order both of magnitude and rarity, has two copies of *Die Coss*, one with each of the title-pages. The old title date is 1553; which indicates that the printing commenced in 1553, and that the colophon fell into 1554.

The first title is—Die Coss Christoffs Rudolffs Mit schönen Exempeln der Coss Durch Michael Stifel Gebessert und sehr gemehrt. Den Inhalt des gantzen Buchs such nach der Vorred. Zu Königsperg in Preussen Gedrückt, durch Alexandrum Lutomyslensem im jar 1553. Here and below I have written ü &c. for *u* &c. with *e* superfixed.

The second title is—Die coss Christoffs Rudolffs, mit schönen Exempeln der Cosz. Durch Michael Stifel Gebessert unnd sehr gemehrt. Den Inhalt des gantzen Buchs such nach der Vorred. M.D. LXXI

The work is in quarto, and the colophon is—Gedrückt zu Königsberg in Preussē, durch Alexandrum Behm von Luthomisl vollendet am dritten tag dess Herbstmonats, Als mannzalt nach der geburt unsers lieben herrn Jesu Christi 1554.

I extract from the work some prominent parts relating to + and - .

(Fol. 62, verso) Christoff Rudolff

Des fünfft Capitel Ist von dem Algoritmo der Coss, so zu Latin genennet wirt De additis et diminutis integrorum. Das ist, von zugesetzten und abgezognen zalen Wirt der zusatz vermehrt bey dem zeychen + bedeut Plus. Der abzug bey dem zeychen—bedeut Minus.

(Fol. 64 verso)

Wen̄ du summirst	{	$\left\{ \begin{array}{c} + \\ - \end{array} \right\}$	Mit	$\left\{ \begin{array}{c} + \\ - \end{array} \right\}$	addir und schreyb	$\left\{ \begin{array}{c} + \\ - \end{array} \right\}$
		$\left\{ \begin{array}{c} + \\ - \end{array} \right\}$		$\left\{ \begin{array}{c} - \\ + \end{array} \right\}$		Subtrahir ein zal vō der andern, schreib zū ubrigen das zeichē der grössern

(Fol. 68, 69)

Dise figur lernt wie man sich im multipliciren halten soll bey den zeychen + und - .

Seind die zeychen	{	Gleych, als wann du multiplicirst + mit + Oder - mit - .	{	Schreib zum pro- duct das zeychen	$\left\{ \begin{array}{c} + \\ - \end{array} \right\}$
		Ungleych, als wenn du multiplicirest + mit - Oder - mit +			$\left\{ \begin{array}{c} + \\ - \end{array} \right\}$

There are later editions of Rudolff which I need not notice. I may conclude* by reminding the reader that nothing in this paper is finally asserted. I am prepared to think it quite possible that + and —, as *more* and *less*, may be fished out of a manuscript of the twelfth or thirteenth century.

A. DE MORGAN.

UNIVERSITY COLLEGE, LONDON,
November 5, 1864.

* In the paper which follows will be seen a valuable collection of notes on the oldest printed books of arithmetic, by my old pupil, Prof. Clifton. The instances given bring out the tendency towards symbolism of the period immediately preceding the publication of Widman's book, and at the same time give no sign of the existence of + and — as *more* and *less*. What is said on the shape of the sign + reminds me to state that in Widman the vertical bar is the shorter, but not so much shorter as to strike a modern eye as peculiar. There are two sizes of type, and I suspect the larger size to be really a woodentype. The shortness of the vertical bar is exceedingly common in the 16th and 17th centuries. I possess Viviani's copy of Bombelli, with his marginal notes, in which it is conspicuous.

The work of Mader referred to by Zedler is 'Centuria Scriptorum qui in celeberrimis præsertim Lipsiensi.....Academiiis usque ad annum 1515 floruerunt. Ab auctore anonymo concinnata nunc vero edita a Joach. Joh. Madero,' Helmst. 1650, 4to. See also Gretschel's 'History of the University of Leipsic,' p. 195. (*Communicated*).

While this paper was going through the press M. Berjeau pointed out to me Weyermann's *Nachrichten von gelehrten... aus Ulm*, Ulm, 1798 (second volume, 1829, with a longer title), 8vo. In the second volume is a life of J. Widman the *physician* (born at Möchingen). It is stated that in 1506 he was appointed state-physician at Ulm, and that he died at Pforzheim, Dec. 31, 1524, in the cathedral of which place he and his wife were buried; he having lived there *einige Jahre* before as private physician to the Margrave of Baden. When he moved from Ulm to Pforzheim of course is not known. Now the edition of 1508 of the work of Widman the *arithmetician* was printed at Pforzheim; and that of 1519 at Hagenau, less than fifty miles off, a small matter between author and printer in those days. It looks unlikely that any one should have printed at Pforzheim in 1508 unless he lived there. Ulm had seven or more printers in 1473—1500; but we know of no press at Pforzheim before 1500, when John of Altenstaig—a neighbour, as appears by the name—printed his *Vocabularius*

there at the press of T. Anselm of Baden, another neighbour. We may suspect that the Pforzheim press was a tardy and a local institution: so that though Widman tried it in 1508, because he resided in the town, he preferred in 1519 to print at Hagenau, the nearest place where there was an older press, printing having there commenced in 1489.

Some presumption arises that the arithmetician was in or near the west of Wirtemberg in some part of 1508—1519; which we know to be true of the physician; and something is added to the presumption that they were the same persons. If we could fix the physician at or near Pforzheim in 1508, the presumption would be much augmented.

Sept. 16, 1865. A few days ago Mr Inglis caught sight of the name of Widman in an old grammar upon a stall. This book is the second edition of the *Grammaticæ Institutiones Jacobi Henrichmanni* with the *Ars condendorum carminum* of Henry Blebelius attached; Leipzig, 1510, 4to. Blebelius gives a letter from himself headed 'Ad viros...Joannem Salicetum alias Widman, ejusque filios utriusque juris doctores. [Here the page ends with a full stop, and the sentence is continued on the next page] Beatum ducalem senatorem atque Ambrosium juris civilis ordinarium professorem Tubinge.' The letter is to the physician, who is addressed as 'te parentem medicum apud Suevos primarium.' There is also a copy of verses by Blebelius 'Ad...doctorem Joh. Salicetum alias Widman, Principis nostri Udalrici medicum, Carmen Henrici Blebelii de nuptiis filie ejus cum doctore Jacobo Kurser cancellario Badensi. Anno M.D. iiii.' Since Widman married his daughter to a Baden official in 1504, the chances of his having been in the neighbourhood of Pforzheim earlier than 1508 are increased. It must be said that neither the letter nor the verses give any hint of Widman Salicetus being an arithmetician: but on the other hand we know that a scholar and poet-laureate of that day would make small account of a vernacular book on commercial reckoning, even if he had ever heard of it. The question of one Widman or two must remain unsettled for the present.



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